THE SUGAR MARKET IN COLOMBIA AND ITS INTERACTIONS WITH THE USA MARKET*

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Resumen
En este artículo se propone un modelo de equilibrio parcial para entender el comportamiento del mercado del azúcar en Colombia y sus interacciones comerciales con la economía norteamericana. Se evalúan los posibles efectos de un TLC con Estados Unidos sobre el mercado, al igual que la incidencia de otros factores internos y externos. Se prueba que el impacto de un TLC entre Colombia y Estados Unidos depende del balance de dos fuerzas contrapuestas: el incremento en la proporción de la cuota que corresponde a las exportaciones colombianas, y la baja progresiva que se ha venido presentando en la cuota total de la economía norteamericana.

Palabras Clave
Mercado el azúcar, comercio internacional, tratado de libre comercio, cuotas de importación.

Abstract
In this paper it is proposed a partial equilibrium model for the sugar market of Colombia to understand its trade relations with the North-American economy. The possible effects of a Free Trade Agreement and of other internal and external factors are examined modelling the functioning of the North-American sugar market. It is proved that the impact of a FTA between Colombia and the USA depend of two forces: the impact of the increase in the proportion of the quota affecting the Colombian exports, and the effect of progressive decreases in the value of the total North-American quota.

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Key words
Sugar market, international trade, free trade agreement, import quotas.

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The sugar market in Colombia and its interactions with the USA market

In this paper a partial equilibrium model is used to capture some important features of the sugar market in Colombia. This model is partially inspired by the excellent work of Tatiana Prada\(^1\), in which for simplicity it is assumed that the sugar market is competitive. One of the main contributions of the present work is the introduction of the demand-side analysis in the model and the connection made with the North American sugar market. To center the analysis in the trade of sugar and for simplicity, the impact of the production of ethanol for the sugar industry is not considered. Future papers in the field should fill this gap. The paper is organised as follows. First, the market is modelled in autarky. This constitutes a very simple and ideal approximation, in which the real international context of the market is ignored. However it represents a natural point of departure for further analysis. Then, a simple model for the USA's sugar market is presented, as a key piece to explain the complete model for the open market in Colombia. Next, the market is modelled in an international context including the North American economy and the rest of the world. Finally, conclusions are stated.

1. The Model in Autarky

A. Assumptions

It is supposed that the sugar market in Colombia is competitive, although the number of firms—i.e. sugar mills—\((n)\) is given and small. That is because there are strong barriers to the entry of new firms derived, partly because of the difficulty of having large extensions of arable land, and partly because of the heavy machinery required to produce sugar. As soon as the marginal cost curve is always over the average cost curve, firms will obtain positive profits. Following Prada (2004) it is supposed that the technology of the typical sugar mill is given by a quasi-fixed factor function as bellow

\[
q_i = \min \left\{ \frac{H_i}{\lambda}, f(K_i) \right\}; \quad f(K_i) = \alpha K_i^\beta, \quad 0 \leq \beta \leq 1, \quad \alpha > 0, \tag{1}
\]

where \(q_i\) is the quantity of sugar produced by the sugar mill \(i\), \(H_i\) is the quantity of sugar cane used in the milling, and \(K_i\) denotes the quantity of a compound factor including capital and labor; \(\lambda\), the technical coefficient, measures the quantity of sugar cane required to produce one unit of sugar, and \(\alpha\) and \(\beta\), the parameters associated to the compound factor, explain the concavity of the function.

The final consumers of sugar have quasi-lineal preferences of the type

\[1. \text{ See Prada (2004)}\]
where $y^d$ represents the quantities of a compound Hicks-Marshall good (the rest of the goods) that a representative agent consumes, $q^d$ indicates the quantities of sugar that he demands, $\gamma$ is a parameter capturing the bias in the expenditures of $q^d$, and $\theta$ is a parameter related to the price-elasticity in the demand for sugar; indeed $\varepsilon_p = 1/\theta < 1$ corresponds to this elasticity (at an individual and an aggregate level), so that it is supposed that the demand for sugar is inelastic in prices. This is reasonable when speaking of a basic good. Many empirical works for different countries have proved that the demand for sugar tends to be inelastic in prices\textsuperscript{2}. There are two types of consumers: Type 1 consumers who have low levels of income and type 2 consumers who have high levels of income. So that, the total number of consumers (the total population ($N$) can be divided between these two types

\[ N = N_1 + N_2 \]  \hfill (3)

Therefore, the equilibrium condition for the sugar market is given by

\[ nq_i = N_1 q_1^d + N_2 q_2^d, \]  \hfill (4)

where $q_1^d$ and $q_2^d$ are the quantities of sugar demanded for the type 1 consumers and the type 2 respectively. The right side of (4) represents the aggregate demand of sugar, and the left side represents the aggregate supply.

\[ \textbf{B. Production} \]

Given the technology of quasi-fixed factors the total cost function of the firm is given by

\[ C(q_i) = R\lambda q_i + r \left( \frac{q_i}{\alpha} \right)^{1/\beta}, \]  \hfill (5)

where \( R \) represents the price of the sugar cane and \( r \) the price of the compound input. It can be proved that the average cost function is always under the marginal cost function except of the origin, where the two curves coincide. That is because of the concave form in which the compound factor enters into the production function. The chart 1 shows the average and the marginal costs functions together.

**Chart 1.** The Average and Marginal Cost Functions of the Domestic Sugar Mills

Consequently, the problem of the typical firm is

\[
\max_{q_i} \Pi_i = pq_i - C(q_i).
\]  

(6)

The price policy of the firm implies that in the optimum

\[
p = R\lambda + \left( \frac{r}{\beta} \right) \cdot \alpha^{-1/\beta} \cdot q_i^{1/\beta-1};
\]  

(7)

Hence the supply curve of the firm is

\[
q_i(p) = \left[ \left( \frac{\beta}{\lambda} \right) \alpha^{1/\beta} \left( p - R\lambda \right) \right]^{\beta/(1-\beta)}.
\]  

(8)

an increasing concave function of \( q_i \), that is positive if and only if \( p > R\lambda \). In equilibrium the profit function of the firm is given by
It is an increasing-concave function of the market price.

\[ \Pi_i(p) = (1 - \beta)[(\beta/r)^\beta \alpha(p - R\lambda)]^{(1-\beta)} > 0; \]  

\[ (9) \]

C. Demand

The problem to be solved by the representative consumer is

\[ \max_{q^d, y^d} U(q^d, y^d) = y^d + \gamma \left( \frac{q^{d(1-\theta)} - 1}{1 - \theta} \right) \]

s.a. \[ y^d + p \cdot q^d = I, \]

where \(I\) denotes the income of the representative consumer. Due to the linear component of the utility function, a classical optimization to solve this problem cannot be used, because it can be obtained corner solutions. Instead, it can be used the Kuhn-Tucker theorem. The Kuhn-Tucker conditions are:

\[ \frac{\gamma}{q^d} - \lambda p \leq 0, \quad q^d \geq 0 \]
\[ 1 - \lambda \leq 0, \quad y^d \geq 0 \]
\[ I - y^d + p \cdot q^d \geq 0, \quad \lambda \geq 0, \]

in each case with complementary slackness. With two non-negative variables and one restriction of inequality, there are 8 possible patterns of equations and inequalities. Nonetheless, it can be proved that there are only two types of solutions:

\[ \text{Type 1: } q^d = I_1/P, \quad y^d = 0 \text{ if } I \leq \gamma^{1/\theta} P^{1-1/\theta}, \]
\[ \text{Type 2: } q^d = \gamma^{1/\theta} P^{-1/\theta}, \quad y^d = I_2 - \gamma^{1/\theta} P^{1-1/\theta} \text{ if } I > \gamma^{1/\theta} P^{1-1/\theta}, \]

where the index for the variable of income denotes the type of consumer. The first type corresponds to a group of consumers with relatively low income. They are poor people, who spent their budget for basic needs—mainly sugar or other similar goods which provide sufficient amount of calories to survive—, because they need calories to survive. For them \( I \leq \gamma^{1/\theta} P^{1-1/\theta}. \) The second type corresponds to another group of consumers with re-

3. Type 1 consumers include the workers of the sugar industry, who have low levels of income in Colombia.
latively high income. It is shown in the equation (14), they consume positive quantities of both goods.

**D. Market Equilibrium in autarky**

With (13) and (14) the aggregate demand for sugar is given by

$$D(p) = N_1(I_1/p) + N_2\left(y^{1/\theta} \cdot p^{-1/\theta}\right);$$

(15)

It can be proved that with inelastic individual demands, the aggregate demand for sugar is also inelastic. From (13) and (14) in (4) we have

$$S(p) = nq_i = N_1(I_1/p) + N_2\left(y^{1/\theta} \cdot p^{-1/\theta}\right)$$

using (8) gives

$$n \cdot \left[\left(\beta / \lambda\right)\alpha^{1/\beta} \left(p - R\lambda\right)\right]^{\beta/(1-\beta)} = N_1(I_1/p) + N_2\left(y^{1/\theta} \cdot p^{-1/\theta}\right)$$

that can be written as

$$n \cdot \left[\left(\beta / \lambda\right)\alpha^{1/\beta} \left(p - R\lambda\right)\right]^{\beta/(1-\beta)} = N\left(\rho(I_1/p) + (1 - \rho)\left(y^{1/\theta} \cdot p^{-1/\theta}\right)\right)$$

(16)

where $\rho$ is the fraction of consumers with low income and $1 - \rho$ the fraction with high income. By using implicit derivation, can be better understood the behaviour of the sugar market in autarky. Graphically:

**Chart 2. The Sugar Market in Autarky**

Source: Author.

**E. Some Results**

Firstly, it can be proved that positive changes in population lead to increases in the equilibrium market price (for (16)) and, therefore, in the quantity of sugar traded in the market, because the population growth shifts the demand curve upwards.
Secondly, as expected, it can be proved that more firms lead to lower prices and increases in the equilibrium quantities of sugar traded, since it shifts the supply curve downwards. So, the more firms in the market are, the less production by each firm due to decreases in prices (see (8)), but this effect is overwhelmed in the aggregate.

Thirdly, an increase in the fraction of consumers with low incomes drops the sugar price in equilibrium, because, although these consumers do not demand the other good, the positive impact they cause in the demand for sugar is slighter than that of the other group of consumers. Remember that hypothetically \( I_1 < \gamma^{1/\theta} p_2^{1-1/\theta} \) and \( I_2 > \gamma^{1/\theta} p_2^{1-1/\theta} \), so that \( I_2 > I_1 \).

Fourthly, evidently increases in the type 1 consumer's income, ceteris paribus make the demand for sugar shift upwards implying a higher demand for sugar, and therefore higher prices. Finally, one increase in the bias of expenditures of \( q^d (\gamma) \) leads to higher prices as well, due to the rising demand for sugar.

Hence, from the implicit function theorem it can be defined the following implicit function for \( p \):

\[
p = p(N, n, \rho, I_1, \gamma); \quad \frac{dp}{dN} > 0, \quad \frac{dp}{dn} < 0, \quad \frac{dp}{d\rho} < 0, \quad \frac{dp}{dI_1} < 0, \quad \text{and} \quad \frac{dp}{d\gamma} > 0. \quad (17)
\]

**F. One Special Case**

With perfect complement factors the total cost function becomes linear—the technology becomes a Leontief production function—, in which the marginal cost is given by \( R \lambda + \frac{r}{\alpha} \). Consequently, the market equilibrium implies that for a specific number of firms, \( n \), the price must equal the marginal cost. Hence the equilibrium quantity of sugar depends on the demand exclusively, and the firms obtain zero profits. Graphically:

**Chart 3.** The Market in Autarky with Perfect Complement Factors

Source: Author.
In this case the market equilibrium is given by

\[ q_i \cdot n = N \left( \rho \cdot I_1 \left( R \lambda + \frac{r}{\alpha} \right)^{-1} + (1 - \rho) \cdot \gamma^{1/\theta} \left( R \lambda + \frac{r}{\alpha} \right)^{-1/\theta} \right). \]

Solving for \( q_i \) gives

\[ q_i = \frac{N}{n} \left( \rho \cdot I_1 \left( R \lambda + \frac{r}{\alpha} \right)^{-1} + (1 - \rho) \cdot \gamma^{1/\theta} \left( R \lambda + \frac{r}{\alpha} \right)^{-1/\theta} \right), \quad (16') \]

So that, though equilibrium quantities are not affected by supply factors, it depends on the demand factors in the same way as the general case with quasi-fixed factor’s technology. It is worth to be mentioned that \( n \) has a negative impact on the firm’s supply, however it does not affect the aggregate supply.

2. The Sugar Market in the USA

USA is a net importer of sugar. Generally the internal price there, \( p^* \), is lower than the international price, \( p^I \). The North American sugar policy is based on a system of quotas and a Sugar Loan Program (SLP) for the maintaining of prices (Beguin et al. 2001). Therefore, the relevant internal price is not the market price but the quota’s price. Furthermore, there is established a minimum price \( p^{\min}_c \) below from which the quota price shouldn’t fall. So that, if eventually the quota price \( p^*_c \) were lower than the minimum, the government should have to finance the losses in profits for the firms –which are:

\[ \left( p^{\min}_c - p^*_c \right) \cdot q_i (p^{\min}_c) + p^*_c \cdot \left( q_i (p^{\min}_c) - q_i (p^*_c) \right) \] through the SLP.

A. Assumptions

Due to the less available land and the worse climate conditions, it is supposed that there are strong decreasing returns to scale in the production of sugar and production technology is given by a Cobb-Douglas function as follows

\[ q^*_i = H^*_i K^{*b}_i, \quad (18) \]

where \( 0 < a < 1, \ 0 < b < 1, \ \text{and} \ a + b < 1 \). Indeed, as the decreasing returns to scale are strong, it is useful to assume that \( a + b < 0.5 \). Here \( H^*_i \) represents the quantity of red beet, corn, or sugar cane, and \( K^*_i \) indica-
tes the quantity of a compound factor— analogous to that for the domestic firms— used by a representative firm or sugar mill. It is worth noting that this technology is more flexible than the one for the domestic economy, because there is a moderate degree of substitution between the two factors of production—i.e. the elasticity of substitution between them is 1. This makes sense, if we take into account the frequent technological advances in the North American agriculture.

North American consumers have identical preferences to the domestic consumers, therefore we have

\[ U(y^d, q^d) = y^d + \gamma \left( \frac{q^d - 1}{1 - \theta} \right), \quad \gamma > 0, \theta > 0, \]  

Consequently there is just one group of consumers: II type consumers.

The market equilibrium is given by

\[ n^* q_i^* + M^* = N^* q^d, \]  

where \( M^* \) indicates the imports of sugar in the North American Economy, \( n^* \) is the total number of firms, that is suppose to be given, and \( N^* \) the total number of consumers.

### B. Production

From (18) by minimizing costs, the total cost function of the typical firm can be deduced as

\[ c(q_i^*) = \mu q_i^* \frac{1}{a+b}, \]  

where \( \mu \equiv \left( \frac{a}{b} \right) \frac{b}{a+b} + \left( \frac{a}{b} \right) \frac{-a}{a+b} \left[ R^* \frac{a}{a+b} r^* \right] \]

\( R^* \) is the price of the red beets, the corn or the sugar cane in the USA and \( r \) the price of the compound factor in that country. This is a convex function. The average cost function is a convex function of \( q_i^* \) too. Moreover, it can be proved that the marginal cost function is always above the average cost function, implying firms to obtain positive profits. Graphically
Chart 4. The Average and Marginal Cost Functions of the North American Sugar Mills

![Chart 4](image)

Source: Author.

The first order condition for the maximization of firm’s profits enables us to get the supply curve of the typical firm:

\[ q_i^* (p^*) = \mu^* \frac{a+b}{1-a-b} \]  

(22)

The price elasticity for the supply of each firm is given by

\[ \eta_{ip}^* = \frac{a + b}{1 - a - b} \]  

(23)

which is lower than one and positive by the assumptions of the model.

The aggregate supply curve is

\[ S^* \equiv n^* q_i^* (p^*) = n^* \mu^* p^* \frac{a+b}{1-a-b} \]  

(24)

So that, the price elasticity for the aggregate supply curve equals the price elasticity for the individual supply curve.
\[ \eta_p^* = \frac{a + b}{1 - a - b} \]  

C. Demand

The problem to be resolved by the representative consumer in the North American economy is

\[ \max_{q^*, y^*} U(q^*, y^*) = y^* + \gamma \left( \frac{q^{1-\theta}}{1-\theta} - 1 \right) \]

s.a. \[ y^* + p^* q^* = I^* \]

where \( I^* \) represents the income of the representative consumer and \( \theta > 1 \) is a parameter related to the price elasticity for the consumption of sugar. Using The Kuhn-Tucker Theorem and noting that all the North American consumers have a relatively high rent, so that \( I^* > \gamma^{1/\theta} p^{*1-1/\theta} \), we have

\[ q^* = \gamma^{1/\theta} p^{*-1/\theta}, \quad y^* = I_2 - \gamma^{1/\theta} p^{*1-1/\theta} \]  

Therefore the aggregate demand curve for sugar is

\[ D^*(p) = N^* q^* d = N \cdot \gamma^{1/\theta} p^{*-1/\theta} \]  

It can be proved that the price elasticity of aggregate demand, likewise the price elasticity of the individual demand, is \( \varepsilon_p^* = \frac{1}{\theta} < 1 \).

D. Market Equilibrium in Autarky in the USA

By making \( M^* = 0 \) in (20) and replacing (24) and (27) we have

\[ n^* \mu^* p^{*a+b} = N^* \cdot \gamma^{1/\theta} p^{*1-1/\theta} \]
Solving for \( p^* \) gives

\[
 p_0^* = \left( \frac{n^* \mu^*}{N^* \gamma^{1/\theta}} \right)^{(1-a-b)\theta / ((1-\theta)(a+b)-1)}
\]  

(29)

Replacing in (8) it turns out

\[
 q_0^* (p_0^*) = \mu^* \left( \frac{(a+b)-1}{(a+b)(1-\theta)-1} \right)^{(a+b)\theta / ((1-\theta)(a+b)-1)} \left( \frac{n^*}{N^* \gamma^{1/\theta}} \right)\theta
\]

(30)

Furthermore, it can be proved that the firm’s profits are positive in equilibrium and equal to

\[
 \Pi_{j0} = \mu^* \left( \frac{(1-(a+b))(\theta-1)}{(a+b)(1-\theta)-1} \right)^\theta \left( \frac{n^*}{N^* \gamma^{1/\theta}} \right)^\theta \cdot (1-(a+b)) > 0, \text{ since } a + b < 1/2.
\]

Graphically

**Chart 5. USA’s Sugar Market in Autarky**

Source: Author.
E. Some Results

An increase in the amount of firms reduces the equilibrium price because it shifts the aggregate supply curve downwards as shows (30), although it leads to a decrease in the supply of the individual firm. As a result of that, the more is the competition among firms, the less are their profits.

On the other hand, increases in the number of consumers lead to higher prices and greater sales of sugar in equilibrium (see equations (29) and (30)), since it shifts the aggregate demand upwards. The profits of the firms tend to grow, as more quantities of sugar are sold by each firm.

Finally, increases in the bias on the expenditures of sugar have the same impact as the increases in the number of consumers, because it affects the demand positively.

F. International Trade and Trade Policy

In this section the USA’s sugar market is opened. Generally the international price is lower than the domestic price, so that \( p^I < p^* \). As a result of U.S. intervention policies, sugar prices in the United States were historically higher than in the rest of the world (Heboyan et al 2001, 4). Nonetheless Tariffs fixed by the North American government are not considered for simplicity.

Free Trade

Given the assumption above the free trade market equilibrium requires that

\[ n^* q_i^* (p^I) + M^* = N^* q^*^d, \]  

(30)

with (22) and (27) evaluating in \( p^I \) and solving for \( M^* \), we have the import function of sugar for the USA.

\[ M^* (p^I) = N^* \gamma \Gamma(p^I) - n^* \mu^* p^I \frac{a+b}{1-a-b}. \]

(31)

Without doubt this is a decreasing function of the international sugar price. It can be proved that the profits of the firms are positive in free trade an equal to

\[ \Pi_{ii} (p^I) = \mu^* p^I \frac{1}{1-(a+b)} \cdot (1-(a+b)) > 0. \]

(32)

Besides, it can be assumed that the world exports of sugar are an increasing function of the international price of sugar and of the out of the quota’s North-American exports. That is because the North-American economy
acts as a big economy. However, this curve is not modelled and $p^f$ is treated as given so far.

Import Quotas
The North-American government establishes an import quota in order to reach a higher production of sugar and to maintain a higher price than the international one. The amount of the quota, $M_c^*$, is supposed to be given. The quota equilibrium can be obtained substituting $M_c^*$ into (31). We get

$$M_c^* = N^* \gamma^{1/\theta} p_c^{* -1/\theta} - n^* \mu^* p_c^{* a+b \over 1-a-b}.$$  

(33)

Even though it is impossible to clear $p_c^*$, it can be analysed implicitly. By using the implicit function theorem, it can be proved the following:

First, a lower quota level leads to a higher quota price, because the pre-existent demand excess pushes the price upwards until reducing it to the new level of the quota imposed.

Second, given the level of the quota, a growing total population implies a higher quota price, because it leads to a larger level of demand and, consequently, to a higher potential demand excess.

At last, more firms (a higher $n^*$) generate a larger aggregate supply and, this way, a lower level of potential demand excess. Since the level of the quota is given, the quota price tends to fall until the new position of the aggregate supply curve is matched to the pre-established level of the quota. The opposite happens with a higher $\gamma$, because it pushes the demand upwards.

Taking into account the latter, and by using the implicit function theorem, we can define an implicit function for $p$ with respect to $M_c^*$, $N^*$, $n^*$, $\gamma$. This function is written as

$$p_c^* = p_c^*(M_c^*, N^*, n^*, \gamma); \quad {dp_c^* \over dM_c^*} < 0, \quad {dp_c^* \over dN^*} > 0, \quad {dp_c^* \over dn^*} < 0, \quad {dp_c^* \over d\gamma} > 0.$$  

(34)

The profits of the sugar mills are higher with the quota, because the quota price is higher than the market price. These are

$$\Pi_{i0}(p_c^*) = \mu^* p_c^{* n^* (1-(a+b))} \cdot (1-(a+b)) > 0,$$

(35)
It can be proved that the elasticity of the quota price with respect to its level is given by

\[- \frac{dp_c^*}{dM_c^*} \cdot \frac{M_c^*}{p_c^*} = \varepsilon_{pM_c} = \frac{N^* \cdot \gamma^{1/\theta} p_c^{-1/\theta} - n^* \cdot \mu^* \cdot p_c^* \eta_{r}^*}{1/\theta \cdot N^* \cdot \gamma^{1/\theta} p_c^{-1/\theta} + \eta_{r}^* n^* \cdot \mu^* \cdot p_c^* \eta_{r}^*} > 0.\] (36)

This elasticity is lower than 1 for not too large values of $\theta$.

3. Bilateral Trade between Colombia and the USA

A. Production
The domestic economy costs are given by equation (5)

\[C(q_i) = R\lambda q_i + r \left( \frac{q_i}{\alpha} \right)^{\frac{1}{\beta}},\]

where $q_i$ is the total production of sugar of the representative firm; $x_i^*$ are the domestic quota-exports to the USA; while $\phi$ is the fraction of the out of quota production sold by each firm on the internal market of sugar, $1 - \phi$ is the fraction sold by each firm abroad. These fractions are supposed to be given for a representative firm$^4$.

\[\max_{q_i} \Pi_i = p\phi^* (q_i - x_i^*) + p_j (1 - \phi^*) (q_i - x_i^*) + p_c \cdot x_i^* - R\lambda q_i - r \left( \frac{q_i}{\alpha} \right)^{\frac{1}{\beta}},\] (37)

$^4$ A more realistic model should endogenize $\phi$ in function of the particularities of each firm and the functioning of the Sugar Price Stabilization Fund. Nonetheless in this model it is supposed for simplicity that all the domestic firms are identical. In Prada (2004) the same parameter is treated as exogenously given too. However, Prada omits the fact that with the definition of $\phi$, the domestic sales are not given by $\phi Q$ but for $\phi (Q - Q_{USA})$ (in her own notation), to the extend that $Q_{USA}$ enters in the total cost function as a part of $Q$. 

\[4\, B.\, Trade\, between\, Colombia\, and\, the\, USA\]

\[A.\, Production\]

The domestic economy costs are given by equation (5)
Since only a fraction of the North-American quota is assigned to the domestic exports, the per firm exports of sugar from Colombia can be written as a function of the total quota in the USA and \( n \); these are

\[
x_i^* = \frac{\psi M_c^*}{n},
\]

where \( \psi \) is the fraction of the total quota corresponding to the Colombian economy.

As \( M_c \) and \( \psi \) are policy variables for the North-American economy, they could not be choice variables for the domestic firms. Therefore, they are considered exogenously given for each firm in the same way as prices. Firstly, as the firms are price takers, the domestic (internal) price is supposed to be given for them. Secondly, as stated above, the quota price in the USA is an inverse function of the level of the quota in this country (see equations (34) and (36)). Thirdly, the international price is determined abroad. It positively depends on the level of the total quota because this latter determines the level of the North-American exports, which has some influence on the level of the international supply of sugar. A lower level of quota induces a higher supply of sugar in the North-American sugar mills, and so a higher level of exports from this country to the rest of the world. Since the USA plays as a big country in the world sugar market, a higher level of their exports shifts the world supply of sugar downwards, giving a negative impact on the international price; the opposite happens when the level of the quota grows. So that we have

\[
p_i = p_i(M_c^*), p_i'(M_c^*) > 0
\]

Tanner (1994) has given conclusive empirical support of this phenomenon since many years.

During the years 1986–1988, the internal production of sugar (in USA) grew. Consequently, the sugar quota –in absolute terms, as a percentage of the internal production, and as a percentage of the internal commercialization– decreased dramatically. ... Since these years were characterized by low international prices of sugar, and not by an extraordinary high production, it could be thought that the decrease of the North-American quota contributed to the movements in the world price of sugar (Tanner, Op. cit, 96)\(^5\).

\(5\). Translated by the autor.
Hence the quota-elasticity of the international price should be positive but not larger than 1, because, although USA is one of the principal producers of sugar in the world, their exports only represent a relatively small fraction of the total exports of the world. For example, in 2008 they represented only a level of 4.6% of the total world production of sugar (ASOCAÑA, 2009). This elasticity can be expressed as \( \xi = \frac{d \Pi_i}{d M^*_c p_i} \). So the problem of the typical firm can be rewritten as

\[
\max_{q_i} \Pi_i = p \phi \left( q_i - \frac{M^*_c}{n} \right) + p_i (1 - \phi) \left( q_i - \frac{M^*_c}{n} \right) + p_c \cdot \frac{M^*_c}{n} - R \lambda q_i - r \left( \frac{q_i}{\alpha} \right)^{1/\beta}, \tag{40}
\]

The first order condition to the solution of this problem is given by

\[
\frac{\partial \Pi_i}{\partial q_i} = p \phi + p_i (1 - \phi) - R \lambda - (r/\beta) \cdot q_i^{1/\beta - 1} \alpha^{1/\beta} = 0, \tag{41}
\]

so that

\[
p \phi + p_i (1 - \phi) = R \lambda + (r/\beta) \cdot q_i^{1/\beta - 1} \alpha^{1/\beta}; \tag{42}
\]

The left side of this equation can be interpreted as a proxy of the weighted average price corresponding to the Sugar Price Stabilization Fund (SPSF). This price can be denoted as

\[
p \phi + p_i (1 - \phi) = pp \tag{43}
\]

Hence, the total supply function of sugar of the typical firm is

\[
q_i(pp) = \left[ (\beta / r) \alpha^{1/\beta} (pp - R \lambda) \right]^{(1-\beta)/(1-\beta)}, \tag{44}
\]

an increasing-concave function of \( p, P_i, \) and \( pp \). The profit function of the firm is given by

\[
\Pi_i(pp) = (1 - \beta) \left[ (\beta / r) \alpha (pp - R \lambda) \right]^{(1-\beta)/(1-\beta)} + (p_c - pp) \frac{\psi M^*_c}{n} > 0; \tag{45}
\]

---

6. See Prada (2004) for a complete explanation of the working of this system.
an increasing-concave function of \( p, P_i, \) and \( pp \). With some algebra it can be proved that

\[
\frac{d \Pi_i}{dp} = \phi \left\{ q_i - \frac{\psi M^*_c}{n} \right\} > 0. \tag{46}
\]

This expression is positive because \( \frac{\psi M^*_c}{n} \) represents only a fraction of \( q_i \).

\[B. \ Demand\]

As defined above the aggregate demand of the domestic consumers is given by

\[D(p) = N_1 \left( I_i / p \right) + N_2 \left( y^{1/\theta} \cdot p^{-1/\theta} \right)\]

\[C. \ Market \ Equilibrium\]

The equilibrium condition requires the aggregate supply to be equal to the internal demand plus the exports;

\[n q_i (pp) = D(p) + X. \tag{47}\]

The demand of exports can be written as partly North-American quota and partly the demand of the rest of the world, that is

\[X = \psi M^*_c + X^i, \tag{48}\]

where \( X^i \) are the demand of exports from the rest of the world (including the out of quota North-American exports), which are considered exogenous.

\[n \cdot \left[ (\beta / \lambda) \alpha^{1/\beta} (pp - R \lambda)^{\theta/(1-\beta)} \right] = N \left( \rho (I_i / p) + (1 - \rho) \left( y^{1/\theta} \cdot p^{-1/\theta} \right) \right) + \psi M^*_c + X^i. \tag{49}\]

This is a non linear function in \( p \), whose analytical solution is impossible to find. But it can be analysed implicitly.

\[G(p, \cdot) \equiv 0 \] the following general function in \( p \) and the rest of exogenous variables and parameters of the model
D. Results

The effect of changes in $N$, $n$, $\rho$, $I_1$, and $\gamma$ on $p$ is the same as in a closed economy. This was already explained in 1.E. Changes in these variables affect the profits of the firms only indirectly by changing prices. Therefore, as increases in $L$ and in $\gamma$ generate more profits, increases in $n$ or in $\rho$ have contrary effects.

Likewise, it is important to explain the effect of changes in the variables related to the external factors, $\psi$, $M^*_c$, and $X^I_i$, which are the key factors in the FTA negotiation between Colombia and the United States.

Firstly, consider the effect of changes in $\psi$, the proportion of the total North-American quota oriented to the Colombian exports. By using the implicit function theorem, it can be proved that increases (decreases) in $\psi$ unambiguously lead to increases (decreases) in the price of sugar, because they have strong positive (negative) effect on the aggregate demand of sugar. In order to understand how changes in $\psi$ affect the profits of the firms is good to consider that it has both a direct and an indirect effect through the changes in the price. The total derivative of the profits of the firms with respect to the changes in $\psi$ is given by

$$G(p, \psi) = \frac{n}{N} \left( \frac{\beta}{\hat{\lambda}} \right)^{1/\beta} \left( pp - R\hat{\lambda} \right) - \rho (I_1/p) - (1 - \rho) \left( \psi \cdot p^{-\psi} \right) - \psi M^*_c - X^I_i \equiv 0 \quad (50)$$

And more specifically

$$\frac{d \Pi_i}{d \psi} = \phi \left( q_i - \frac{\psi M^*_c}{n} \right) \frac{dp}{d \psi} + \frac{dp}{d \psi} \left( p_c - pp \right) \frac{M^*_c}{n} > 0. \quad (51')$$

The above formula (51') is positive, because $\frac{d \Pi_i}{dp}$ and $\frac{d \Pi_i}{d \psi}$ are positive, and the quota price is always higher than the weighted average price. Therefore, as expected, increases in the proportion of the quota assigned to the Colombian exports generate extra profits to the domestic firms.

Secondly, consider the effect of changes in $M^*_c$, the amount of the total quota of the North-American Economy. It can be proved that for low values of $\beta$ and $\xi$ and large values of $L$ as expected the effect of changes in $M^*_c$ on the price of sugar tend to be positive. Consequently, due to the assumptions of the model, $\frac{dp}{dM^*_c}$...
should be positive. It means that when $\xi$, the elasticity of the international price with respect to the total amount of the North-American quota, and the $\beta$ coefficient are relatively low and the total population is sufficiently large as well, the positive impact of an increase in $M^*_c$ on the international price $P_i$ -that stimulates the aggregate supply of sugar- is not strong enough to out-weigh the positive effect that it has in the aggregate demand of sugar. The total impact of changes in the amount of the total quota is given by

$$
\frac{d \Pi_i}{dM^*_c} = \frac{d \Pi_i}{dp} \frac{dp}{dM^*_c} + \frac{d \Pi_i}{dp_i} \frac{dp_i}{dM^*_c} + \frac{d \Pi_i}{dp^*_c} \frac{dp^*_c}{dM^*_c} + \frac{\partial \Pi_i}{\partial M^*_c}.
$$

(52)

It can be proved that

$$
\frac{d \Pi_i}{dp_i} = (1 - \phi) \left( q_i - \frac{\psi M^*_c}{n} \right) > 0
$$

(53)

and that

$$
\frac{\partial \Pi_i}{\partial M^*_c} = \left( p_c - pp \right) \frac{\psi}{n} > 0
$$

(54)

By using algebra we get

$$
\frac{d \Pi_i}{dM^*_c} = \frac{1}{M^*_c} \left( q_i - \frac{\psi M^*_c}{n} \right) \left( (1 - \phi) p_i \xi + \phi p e \right) + \frac{\psi}{n} \left( p_c \left( 1 - \varepsilon_{pM^*_c} \right) - pp \right)
$$

(55)

Its sign is ambiguous in principle, although the last term is the only one that could have a negative impact. For sufficiently large values of $n$ the whole expression is positive, so that, an increase in the level of the quota has a positive impact on profits. It means that the positive impact of

$$
\frac{d \Pi_i}{dp} \frac{dp}{dM^*_c}, \frac{d \Pi_i}{dp_i} \frac{dp_i}{dM^*_c} \text{ and } \frac{\partial \Pi_i}{\partial M^*_c}
$$

is not compensated with the negative effect of

$$
\frac{d \Pi_i}{dp^*_c} \frac{dp^*_c}{dM^*_c}.
$$

Thirdly, using the implicit function theorem it can be proved that an increase in the exports of the rest of the world $(X^i)$ conduces to an increase in the internal price of sugar, and consequently in the domestic supply of sugar. For that, is has a positive impact on the profits of the firms. This effect has been very important for the Colombian sugar industry during this decade: In particular, it can be checked using descriptive statistics that the dramatic fall of the Colombian exports during the last years explain in part the downward trend presented
in the total supply of sugar during this period. Whereas the total supply of sugar fall from 2.741.363 MT in 2004 to 2.036.164 MT in the year 2008, the direct exports of sugar decrease more rapidly from 1.232.782 MT in 2004 to 478.442 MT at the end of that period. The other relevant factor explaining this dramatic trend in the supply of sugar is the growing production of ethanol (Raffo 2009).

Finally, it is important to analyse the effect of changes in the fraction of the out of quota production, sold by each firm on the internal market of sugar \( \phi \). It can be proved that the effect of changes in \( \phi \) on the price of the sugar depends on the relative measure of this price with respect to the international price of sugar. More precisely, one increase (decrease) in \( \phi \) tends to a decrease in the internal price of sugar, if and only if, this price is higher than the international price, i.e.

\[
\frac{dp}{d\phi} \geq 0 \iff p \leq p_i
\]

(56)

That is because an increase (decrease) in \( \phi \) has two different effects on the aggregate supply of sugar: it increases the supply, as it increases \( p\phi \), but it decreases the supply, as it has a negative impact on \( p_i (1 - \phi) \). As \( p\phi + p_i (1 - \phi) = pp \) for (43), it can be inferred that the effect of increases (decreases) in \( \phi \) on the price depends on how it alters \( pp \), the proxy of the weighted average price corresponding to the Sugar Price Stabilization Fund; When \( pp \) increases (decreases), a rise (fall) in the aggregate supply of sugar occurs, and hence a decrease (an increase) in the internal price of sugar.

With respect to the impact of changes of \( \phi \) on the profits, it is important to consider not only its direct impact, but also its indirect impact through the price, so that

\[
\frac{d\Pi_i}{d\phi} = \frac{d\Pi_i}{dp} \cdot \frac{dp}{d\phi} + \begin{vmatrix} \partial \Pi_i \end{vmatrix}_{\partial \phi}
\]

(57)

As the direct impact equals

\[
\frac{\partial \Pi_i}{\partial \phi} = pp \left( q_i - \frac{\psi M_c^*}{n} \right) > 0,
\]

(58)

with (46) and (57), by using some algebra we have

\[
\frac{d\Pi_i}{d\phi} = pp \left( q_i - \frac{\psi M_c^*}{n} \right) \left[ 1 + \eta_{p\phi} \right].
\]

(59)
when \( \frac{dp}{d\phi} \geq 0 \); or
\[
\frac{d \Pi_i}{d\phi} = p \left( q_i - \frac{\psi M^*_c}{n} \right) \left[ 1 - \eta_{p\phi} \right],
\]
(57')
when \( \frac{dp}{d\phi} < 0 \).

Here \( \eta_{p\phi} \) denotes the elasticity of the price of sugar with respect to \( \phi \):
\[
\frac{dp}{d\phi} \cdot \frac{\phi}{p}, \quad \text{if} \quad \frac{dp}{d\phi} > 0, \quad \text{or} \quad -\frac{dp}{d\phi} \cdot \frac{\phi}{p}, \quad \text{if} \quad \frac{dp}{d\phi} < 0.
\]

In relation to the previous equations, and taking into account (56), it is clear that when \( \frac{dp}{d\phi} > 0 \), the impact of an increase in \( \phi \) on the profits is positive, so that
\[
\eta_{p\phi} = \frac{dp}{d\phi} \cdot \frac{\phi}{p} \Rightarrow \left( \frac{d \Pi_i}{d\phi} > 0 \right);
\]
(60)
But when \( \frac{dp}{d\phi} < 0 \), it depends on its level: on one hand when it is elastic \( (\eta_{p\phi} > 1) \), \( \frac{d \Pi_i}{d\phi} < 0 \), because the negative effect on the price is relatively strong, so that it outweighs the positive effect of \( \frac{\partial \Pi_i}{\partial \phi} \); On the other hand, when it is inelastic \( (\eta_{p\phi} < 1) \),
\[
\quad \frac{d \Pi_i}{d\phi} < 0, \quad \text{the negative effect on the price is relatively weak, to outweigh the positive impact of} \quad \frac{\partial \Pi_i}{\partial \phi}.
\]

In the especial case where the elasticity is one, the negative effect of the price –the first term of the right side of (57)– is neutralised by the positive impact of \( \frac{\partial \Pi_i}{\partial \phi} \).

Therefore
\[
\eta_{p\phi} = -\frac{dp}{d\phi} \cdot \frac{\phi}{p} \Rightarrow \left( \frac{d \Pi_i}{d\phi} \geq 0 \leftrightarrow \eta_{p\phi} \leq 1 \right).
\]
(61)
With (56) it follows that

\[ p_i \leq p \Rightarrow \left( \frac{d \Pi_i}{d \phi} > 0 \right); \]  

(62)

when \( \eta_{\phi} = \frac{dp}{d \phi} \cdot \frac{\phi}{p} \), and that

\[ p_i > p \Rightarrow \left( \frac{d \Pi_i}{d \phi} \geq 0 \leftrightarrow \eta_{\phi} \leq 1 \right). \]  

(63)

when \( \eta_{\phi} = \frac{dp}{d \phi} \cdot \frac{\phi}{p} \).

By taking into account the latter results and the implicit function theorem with respect to \( G(p, \cdot) \), we can define an implicit function for \( p \) with respect to \( N, n, \rho, I_1, \gamma, \psi, M^*_c, X^l_i \) and \( \phi \):

\[ p = p(N, n, \rho, I_1, \gamma, \psi, M^*_c, X^l_i, \phi) \]

\[
\frac{dp}{dN} > 0, \quad \frac{dp}{dn} < 0, \quad \frac{dp}{d\rho} < 0, \quad \frac{dp}{dI_1} < 0, \quad \frac{dp}{d\gamma} > 0, \quad \frac{dp}{dX^l_i} > 0, \quad \frac{dp}{d\psi} > 0, \quad \frac{dp}{d\phi} > 0, \\
\frac{dp}{dM^*_c} > 0 \]

for low values of \( \beta \) and \( \xi \) and large values of \( L \), and \( \frac{dp}{d\phi} \geq 0 \leftrightarrow p \leq p_i \).

(64)

D) The Impact of a FTA Between Colombia and the USA

The initial purpose of Colombia in the negotiations of the TLC was to enter to the North American market with a quantity of 500,000 MT, a quantity considerably higher than the currently quota-exports to Colombia. However, after almost 22 months of negotiation Colombia only achieved an increase of 50,000 MT of additional exports to the North-American economy with an annual increase of 1.5% (approximately 750 MT). By contrast, the North-American government demanded free entry for his own production of substitutes of sugar as the glucose syrup, and, in fact, it was dealt with a tariff liberation for the imports of corn syrup during the next 9 years.

Within the WTO distribution of the quota, Colombia had already entered to the North-American market with 25,000 MT. Even though the share of the Colombian exports in the North-American imports is marginal—it represented only 2.3% in 2004—and it has been diminishing during the last years (see Table 1),
Colombia has been the country with the highest quota in the negotiation of the Free Trade Agreements of the USA. Likewise, the total North American quota has been diminishing too, decreasing from 2,100,001 MT in 1996/97 to 1,117,192 MT in 2007/08. This is shown in Table 1 too.

Hence, ceteris paribus the exports of the rest of the world, \( X_i^\prime \), the impact of the FTA between Colombia and the USA in the next years can be understood as the conjugation of two different processes: First, an increase in the share of the Colombian quota in the total quota, and second a continuous decrease in the total quota level of the United States.

So the possible impact of the FTA to the profits of the domestic firms is given by

\[
\Delta \Pi_i = \frac{d \Pi_i}{dM_c^e} \cdot \Delta M_c^* + \frac{d \Pi_i}{d\psi} \cdot \Delta \psi
\]  

Due to \( \Delta M_c^* < 0 \) and \( \Delta \psi > 0 \), and taking into account (51') and (55), we can say that the possible impact of the FTA is ambiguous: while the first term of the right side of (56) is negative, the second term of the right side is positive. From (56) it can be seen that the sign of the changes in the profits of the firms depends of how the proportion of Colombian exports in the total quota changes as the total North American quota diminishes. If \( - \frac{\Delta \psi}{\Delta M_c^e} \cdot \frac{M_c^e}{\psi} > 0 \) is the elasticity of the proportion of the quota that goes to the Colombian exports with respect to the total North-American quota we have

\[
\Delta \Pi_i \geq 0 \iff \frac{d \Pi_i}{dM_c^e} \cdot \frac{M_c^e}{\Pi} \leq - \frac{\Delta \psi}{\Delta M_c^e} \cdot \frac{M_c^e}{\psi}
\]  

Table 1. Imports of Raw Sugar of the United States (1996/97-2006/07, MT)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Quota Colombia (1)</td>
<td>48.690</td>
<td>36.593</td>
<td>25.999</td>
<td>25.274</td>
<td>25.274</td>
<td>25.274</td>
</tr>
<tr>
<td>Total USA quota (1)</td>
<td>2,100.001</td>
<td>1,600.000</td>
<td>1,164.934</td>
<td>1,135.000</td>
<td>1,223.045</td>
<td>1,117.257</td>
</tr>
<tr>
<td>Ratio (1)/(2)</td>
<td>0.02319</td>
<td>0.02287</td>
<td>0.02232</td>
<td>0.02227</td>
<td>0.02066</td>
<td>0.02262</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Origin/Year</th>
<th>2002/03</th>
<th>2003/04</th>
<th>2004/05</th>
<th>2005/06</th>
<th>2006/07</th>
<th>2007/08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quota Colombia (1)</td>
<td>25.273</td>
<td>25.273</td>
<td>30.235</td>
<td>43.121</td>
<td>30.760</td>
<td>25.273</td>
</tr>
<tr>
<td>Total USA quota (1)</td>
<td>1,117.192</td>
<td>1,117.192</td>
<td>1,193.801</td>
<td>1,717.150</td>
<td>1,336.736</td>
<td>1,117.192</td>
</tr>
<tr>
<td>Ratio (1)/(2)</td>
<td>0.02262</td>
<td>0.02262</td>
<td>0.02533</td>
<td>0.02511</td>
<td>0.00230</td>
<td>0.02262</td>
</tr>
</tbody>
</table>

Source: United States Department of Agriculture (USDA)
Hence the FTA would have a positive impact on the profits of the firms, when the percentage changes in the proportion of the Colombian exports in the total quota with respect to the percentage changes in the total North American quota are relatively small.

4. Conclusions

The simple model developed here shows that under perfect competition in the North-American sugar market, the quota price inversely depends on the total level of the quota and the number of firms, but directly on the total population and the preferences parameter $\gamma$, that measures the bias in the consumption of sugar. The profits of the North-American firms depend in the same way on these factors, because there is a positive relationship between the quota price of sugar and the profits of the firms in the USA.

For the Colombian sugar market the model predicts that increases in internal factors such as the total domestic population and the preferences parameter $\gamma$, affect positively the internal price and, therefore, have a positive influence on the profits of the sugar mills. The opposite occurs when there is an increase in the number of firms: it lowers the sugar price equilibrium and therefore induces drops in the profits of the firms.

Furthermore, as presumed, the model shows that increases in the proportion of the total North-American quota assigned to the Colombian exports, lead to increases in the domestic price and thus in the profits of the domestic firms. This is an important trend that will occur as a result of the application of the FTA with the USA. With respect to the consequences of changes in the level of the total quota, it has been shown that under the assumptions of the model, positive (negative) changes in the level of the total North-American quota, lead to higher (lower) internal prices, when the values of $\beta$ and $\xi$ are relatively low and the value of L is relatively large. Therefore under these conditions, increases (decreases) in the total level of the quota should cause increases (decreases) in the profits of the domestic firms.

By the other hand and as expected, the model predicts that there is a positive relationship between the exports to the rest of the world $(X_i')$ and the internal price of sugar and, consequently, between this variable

$$\Delta \Pi_i \geq 0 \iff \frac{q_i - \frac{\psi M^{*}_c}{n}}{n}((1-\phi)p_1 \xi + \phi p_{e,1}) + \frac{\psi M^{*}_c}{n}(p_c (1-\varepsilon_{pm_i}) - pp) \leq \frac{\Delta \psi}{\Delta M^{*}_c} \frac{M^{*}_c}{\psi}.$$
and the profits of the domestic firms. In fact, the dramatic fall in the exports during the last years explains partly the downward trend presented in the total domestic supply of sugar during this period.

So, with the results found it can be proved that the possible effects of a Free Trade Agreement between the USA in the Colombian sugar sector are essentially ambiguous, because there are two kinds of effects that act in opposed directions: for one side, the impact of the increase in the proportion of the quota affecting the Colombian exports: This leads unambiguously to increases in the profits of the firms; on the other hand, the effect of progressive decreases in the value of the North-American quota, that undermines the profits of the firms. Which effect can prevail depends on its strength and on the values of the relevant parameters in the model. Even though the FTA induces a positive impact over the domestic prices and the profits of the firms by way of the agreed increases in the proportion of the USA’s quota going to the Colombian exports, this impact could be neutralized if the total amount of the North American quota continues decreasing in the future.

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